C I G E

CENTRO DE INVESTIGAÇÃO EM GESTÃO E ECONOMIA

UNIVERSIDADE PORTUCALENSE – INFANTE D. HENRIQUE

DOCUMENTOS DE TRABALHO
WORKING PAPERS

Nº 21 | 2011

MULTI CRITERIA DECISION MAKING MODELS: AN OVERVIEW ON ELECTRE METHODS

Antonieta Lima[1], Vasco Salazar Soares [2]

September 2011

[1]Universidade Portucalense Infante D. Henrique, Assistant Professor of the Economics and Management Department, and Ph.D candidate in Management
antonieta_sousa_lima@hotmail.com

[2] ISVOUGA, Professor of the Finance Department , and Universidade Portucalense , Auxiliar Professor of the Economics and Management Department
vascosalazarsoares@gmail.com

TABLE OF CONTENTS:

Multi Criteria Decision Making Models: An Overview On ELECTRE Methods

Abstract

In portfolio analysis, there are a few models that can be used. Therefore, the aim of this paper is to make an overview on multi criteria decision making models, in particular, on ELECTRE methods.

We discuss the different versions of ELECTRE, which exist and why they exist. So, when speaking about ELECTRE methods structure, we have to consider two main procedures: construction of one or several outranking relation(s) procedure, and exploitation procedure. In the exploitation procedure, recommendations are elaborated from the results obtained in the first phase. The nature of the recommendation depends on the problematic: choosing, ranking or sorting. Each method is characterized by its construction and exploitation procedure. For *choice problem*, we can apply ELECTRE I, ELECTRE Iv, and ELECTRE IS; for *ranking problem*, we can apply ELECTRE II, ELECTRE III, ELECTRE IV and ELECTRE-SS; and for *sorting problem* we can apply ELECTRE TRI.

Finally, some failings on ELECTRE methods assumptions are discussed, for instance, rank reversals. So, when analyzing portfolio management decision problem, the literature suggests AHP method and PROMETHEE family.

Key Words: CAPM, decision problem, multi criteria decision making models, ELECTRE family, and ELECTRE rank reversals.

I – INTRODUTION

In portfolio analysis, there are a few models that can be used: classical model and multi criteria modelling approach. Therefore, the purpose of this paper is to provide an overview on multi criteria decision making models, in particular ELECTRE methods. We discuss the different versions of ELECTRE: which exist, why they exist, and for what kind of problematic they exist.

As any other theory, ELECTRE methodology has theoretical failings, for instance, rank reversals. So, as an alternative to ELECTRE family, the literature suggests AHP method and PROMETHEE family.

Concerning classical methodology, Sharpe (1964), Lintner (1965), Mossin (1966), and Markowitz (1952) were pioneers in defining the oldest and the most widely known of all finance models: the Capital Asset Pricing Model (CAPM). The CAPM model, for estimating expected returns, considers the exposure to systematic risk (beta), the only factor that is related to expected returns.

But, as any other theory, the Markowitz (1952) optimization portfolio theory has limitations. As notice by Cohen and Pogue (1967), in first place, the model was studied and implemented for a certain period. However, when market conditions change, these changes must be considered in the model, so that a new efficient frontier could be established. Several authors propose the use of a sensitivity analysis for different estimation and testing periods. Secondly, as time passes and the portfolio is not corrected, the border points are scrolled to directions, eventually, of lower returns or otherwise. Thirdly, this analysis required long times series to be consistent. Thus, in the long term, the expected return could not be stationary, and the model does not answer how to overcome this disadvantage. Finally, there are errors in the input measurement, estimation errors, that have marked impact on the afterwards results (Elton, Gruber and Padberg, 1976).

Differently from Markowitz (1952) classical theory, in the late '80s and early '90s, the development of new operational research techniques, as well as the computer power, enabled new approaches in the optimal portfolio modelling selection: the "Expert Systems” and the Multi criteria modelling approach. This approach is due, among others, to Lee, Kim and Chu (1989) and Shane, Fry and Toro (1987). Slovic (1964) and, Kogan and Wallach (1967) sustain that, this lack of risk measure is symptom of risk multidimensional phenomenon. For example, in portfolio management, a multi criterion modelling provides the methodological basis for solving the multifaceted portfolios selection and build realistic models and processes. They take into account, besides the two basic factors, risk and return (the classic mean-variance model), one number of important additional factors such as market liquidity, PER (price-to-earnings ratio), dividends growth rate, social responsibility, environmental protection, employee welfare, among others.

Multi criteria decision models has been widely used in real-life decision problems, for instance, TOPSIS, AHP, ELECTRE family, PROMETHEE family, ADELAIS and MINORA. Each one of them has distinct characteristics and distinct applications. Within these methods, literature highlights ELECTRE family, distinguished by their performance and problem resolution. As stated by Buchanan, Sheppard and Vanderpooten (1999), "*Experience with the methodology shows that ELECTRE was well received by the decision makers and, importantly, provided sensible and straightforward project rankings.*"

Thus, the paper is organized as follows. Firstly, it is presented a brief history of multi criteria decision making methods. Secondly, the main features of ELECTRE family are presented, and described the different versions existing in the literature according to the three main problematic: choosing, ranking and sorting. Then, some tests to evaluate multi criteria decision models are discussed. Finally, some criticism on ELECTRE methods assumptions is discussed, and the main conclusions are drawn. A bibliography is provided at the end of this paper.

II – THE MULTI CRITERIA DECISION MAKING MODELS

Markowitz (1952), Sharpe (1964) and Tobin (1958) gave the most representative contributions to modern financial theory. They consolidated a new scientific area, within the economy, in which the key concept is the economic study of the capital market. Markowitz (1952) developed the mean-variance model which was a pioneering attempted to focus the demand for risky assets. Miller (1978) and Sharpe (1964), among other contributions, trailed the propositions that characterize the equilibrium of capital market.

The Modern Portfolio Theory has changed the way investors think about their strategies. The theory assumes that financial markets are efficient, meaning that the price of any asset incorporates all the information existent[3]. The main theory's task is to determine the asset rate of return. It is based on

[3] There are three levels of efficiency defined by: Weak Form Efficiency (prices reflect all information contained in past price movements); Semi-strong form (besides the weakness, it also reflects all the other information published); Strong Form (besides the other two forms, reflects all the information that can be gained through analysis of the company and the economy).

the assumptions of CAPM[4] (Capital Asset Pricing Model): perfect capital market, ability to lend and borrow in unlimited amounts to a common risk-free rate, and homogeneity in public expectations. The CAPM provides two basic conclusions. The first, concerns the degree of optimal portfolio diversification in market equilibrium; the second, is about the appropriate measure of risk assets and the relationship with its expected rate of return. However, this model, and like any model, is merely a simplification or abstraction of reality that helps decision making. Although several of the assumptions are questionable, what must be asked is, if in fact, the model predicts and works well, considering that the model refers to expected returns. It doesn't mean that every CAPM's results are true. As already noted, several objectionable features leaded to the proposal of some alternative theories.

Differently from Markowitz (1952) classical theory, we have the multi criteria modelling approach (see Lee *et al*., 1989; Shane *et al*., 1987; Slovic, 1964; and Kogan and Wallach, 1967). A decision problem according to B. Roy's definition (Roy, 1991), is a representation of an element of a global decision. Zbigniew and Watróbski (2008) distinguishes decision alternatives, in particular, on realistic alternatives (corresponding to a project, which implementation is feasible) and on unrealistic alternatives (which can include contradictory goals and can be only used for the discussion). The difficulty when solving multi criteria decision problems, is the requirement of including alternatives' judgments (choice alternatives) from various points of view, which refers to multi criteria judgments (Escobar-Toledo and López-Garcia, 2005).

To do so, Zbigniew and Watróbski (2008) consider that the definition of a decision problem consists into a two-element process, ***(C, θ)***, where ***C*** represents *a set of criteria*, describing relations between properties of decision alternatives and preference levels of considered alternatives; and ***θ*** represents *a set of meta-data of a decision situation*, consisting into the decision maker's expectations about a decision situation.

An analytic task, stated by the analyst and the decision makers, reflect particular aspects of implementation based on possible options (decision alternatives). The fundamental element of the meta-data set θ is the choice of the decision problematic situation according to the following (Roy, 1991):

- *problematic α – the choice problematic* (finding a subset of the set A set which includes only the best solutions),

[4] The main Sharpe's merit was to extend the Markowitz and Tobin optimal portfolio analysis selection to a model of capital market equilibrium. What we now call CAPM is actually a synthesis of contributions from various authors. Almost simultaneous, Sharpe (1964), Lintner (1965) and subsequent Mossin (1966), Fama (1965) gave an important contribution, that take as a starting point Markowitz (1952,1959) and Tobin (1958) works.

- *problematic β – the sorting problematic* (assigning alternatives to defined categories),

- *problematic γ – the ordering problematic* (constructing a ranking of alternatives in the set *A* from the best one to the worst one).

Such an approach only considers a part of the decision process. Applying multi criteria methods, to analyze a decision situation, requires making a deliberate choice of a method suitable for a given decision situation. The goal of the mentioned choice is to find the multi criteria transformation *F* which fulfils, ***F(C, θ)* -> max *u***, where *u* is an indicator of a decision maker's satisfaction measured by his preferences.

This decision process phase, the *exploitation phase*, intends to make a representation of the global preference which is the outcome of a decision maker's expectations (meta-data) and mutual local preferences between particular decision alternatives.

Zbigniew and Watróbski (2008) indicate some methods suggested by the literature and their characteristics[5]:

- **AHP:** Transform subjective decision maker's judgements into ordered criteria weights. The procedure uses decomposition of the problem and comparison matrix of attributes to create a comprehensive estimate of a decision alternative (Saaty, 1980).
- **ELECTRE family:** The outranking is expressed by the credibility index (Roy, 1991, and Figueira, Mousseau and Roy, 2005).
- **PROMETHEE family:** Based on the concepts of pseudo-criterion, elaborates an outranking relation and pair wise comparisons (Brans and Vincke, 1985).

In this sense, multiple criteria decision methodology has been widely used in many real-life decision problems, for instance:

[5] There are other methods that can be applied, so being, **TOPSIS (**Hwang and Yoon, 1981) defined by choosing an alternative with the shortest distance to the ideal solution and the longest distance to the negative ideal solution, **ADELAIS** (Zopounidis, Despotis and Kamaratoy, 1993) provides an extensive data management capabilities and the concerned solution process provides a 'two level' interaction: interactive assessment of the decision maker's utility function and interactive modification of the satisfaction levels, and **MINORA (**Zopounidis, 1992), developed in order to help decision maker, and their evaluation criteria, when selecting assets to obtain the maximization of their utility.

In project selection for Northern Generation, a division of the Electricity Corporation of New Zealand (ECNZ):

Buchanan *et al.* (1999) used ELECTRE III method to rank minor projects for Northern Generation, a division of the Electricity Corporation of New Zealand (ECNZ). The authors choose this method, because it has several unique features not found in any other solution methods, in particular, the concepts of outranking and the use of indifference and preference thresholds.

This study was motivated by the fact that ECNZ, pretended to introduce a more objective (and structured) method for the annual exercise of selecting minor projects to be undertaken (projects selection are conditioned by financial targets). In this sense, considering five projects, the authors assigned to each project, or alternative, a few attributes, which were then related to the criteria.

From the application of ELECTRE methodology, it follows that project nº 3 and project nº 5 were ranked together. However, Buchanan *et al.* (1999) adds that, a sensitivity or robustness analysis to final rankings should be done, for instance, by changing thresholds and weights.

In Mass Transit Systems (MTS-s):

Zak (2005) applied the multiple criteria decision methodology to the decision problems in mass transit systems (MTS-s). He considered three problems of strategic and tactical character: evaluation of the MTS development scenarios – problem I, ranking of the maintenance work, contractors for the MTS renovations project – problem II and selection of the transportation mode for the MTS – problem III. To solve this multiobjective ranking problem, he applied the following methods: ELECTRE, Oreste, Mappac, AHP and UTA. To do so, stakeholder's / decision maker's expectations were analyzed (survey, interviews, family of criteria, among others), and results were compiled. Their features were measured by the comparison of the final rankings generated by different methods, and by the expected final rankings suggested by each of the respondents.

The results suggested that all the analyzed methods have universal character and can be applied to a wide spectrum of multiobjective ranking problems in MTS-s. Specifically, ELECTRE and AHP methods were the most reliable and users' friendly multi criteria decision methods.

In natural resources management:

A. Kangas, J. Kangas and Pykä (2001) applied ELECTRE III and PROMETHE II in natural resources management in Finland (Finnish Forest and Park Service in Kainuu, eastern Finland), because natural resources are synonymous of economic, ecological and socio-cultural sustainability. So, multi objective natural resources management planning and decision support are required. These authors applied ELECTRE III and PROMETHE II because in both methods the number of decision criteria and decision maker may be large, and the uncertainty concerning the values of the criterion variables can be taken into account using fuzzy relations (determined by indifference and preference thresholds). Therefore, the authors considered this feature an important advantage of outranking methods.

In land redevelopment in Heping Harbor Zone (Taiwan):

Huang and Chen (2005) applied ELECTRE II to a case study based on land redevelopment in Heping Harbor Zone (in Taiwan). Considering fast changes in living environment, many cities have placed increasing expectations on land redevelopment to help ease urban planning problems. So, the authors defined six preliminary improvement alternatives according to the collected information.

As notice by the authors, ELECTRE method allows both quantitative and qualitative criteria to be handled. In this sense, normally discordance index is constructed using *Absolute Value of the Maximum Differentiated Performance* (A.V.M.D.P.) to evaluate benchmark as evaluation procedure. Huang and Chen (2005) conducted another benchmark procedure, namely the *Absolute Value of the Sum of Differentiated Performance* (A.V.S.D.P.) procedure. Those two benchmark evaluation procedures represents different decision maker's approaches: A.V.M.D.P. focus on discrepancies in the most important criteria's, and A.V.S.D.P. focus on discrepancies in the overall criteria. The result shows that *alternative one* is the priority alternative as the discordance index evaluation benchmark for both A.V.M.D.P. and A.V.S.D.P. However, when the decision maker prefers to have more than one alternative to be taken into consideration, the alternatives after the "best" first one vary greatly. Taking the second "best" one alternative, A.V.M.D.P. indicates *alternative two* as the only choice, but A.V.S.D.P. indicates *alternative six* as the only choice. Now, taking the *third alternative*, A.V.M.D.P. gives *alternative four* as the best choice, while A.V.S.D.P. gives *alternative four* as the worse alternative.

Considering these results, Huang and Chen (2005) concluded that the two evaluation benchmarks reflect different decision maker judgment and needs criteria. Therefore, if decision makers choose the discrepancy of overall criteria as the screening benchmark for evaluation alternatives, and using the A.V.M.D.P. evaluation benchmark, the screening results would lead to serious errors.

In the choice of construction equipment:

Serdar and Aynur (2009) analyze the advantages of using concrete pumps on machine selection process. They add that concrete pump may improve productivity, may increase the quality of products and services, and may reduce the duration and the cost of the task "pouring concrete". In the long run, this can contribute to the related firms in improving their competitiveness and in outperforming their competitors in the construction industry.

In this sense, Serdar and Aynur (2009) justify the use of ELECTRE III methodology because this technique allows quantitative data to be evaluated together with qualitative data. Gives a ranking order of alternatives rather than presenting only one option, and have flexible feature which, in turn, makes decision-makers feel more comfortable and independent. As limitation, these authors point out that this method only can be used when at least 3 and at most 13 decision criteria are available (Figueira *et al.*, 2005).

To conducted the experiment, the authors established five quantitative criteria's ("selling price", "operating cost per day", "maximum pumping speed", "second hand" and "technical services"), in order to evaluate three different manufactures of concrete pumps (Z-52, X-52 and Y-52).

Considering ELECTRE III methodology, the final result obtained revealed X-52 as being the most suitable concrete pump, followed by Z-52 and Y-52. To test this final result, six independent experimental attempts of sensitivity analysis were made in particular, the author vary each weight of each criterion separately, and the overall findings point out that the original outcome was not considerably changed.

In personnel selection:

Afshari, Mojahed, Yusuff, Hong and Ismail (2010) suggested ELECTRE method to solve personnel selection problem using multi criteria decision making process, applied in the telecommunication sector of Iran. While traditional methods for selection of human resources are mostly based on statistical analyses of test scores that are treated as accurate reflections of reality, modern approaches, however, recognize that selection is a complex process that involves a significant amount of vagueness and subjectivity. The authors firstly use ELECTRE method for pre-ranking personnel; then, after identifying the level of personnel, they apply AHP method when at least one of personnel's grades was placed in the same with another. At the end, all personnel which had been considered were sorted in different level.

The limitation of this author's study, and one of the failings of ELECTRE methodology, is that executives' judgment is ignored during the decision-making process, although some criteria could have a qualitative structure or have an uncertain structure which cannot be measured precisely. In such cases, fuzzy numbers can be used to obtain the evaluation matrix, biasing model results.

III – THE ELECTRE METHODOLOGY (Elimination Et (and) Choice Translating Reality)

Within the models mentioned on the previous chapter, we can highlight the ELECTRE family, from the "European school" which, as stated by Buchanan *et al.* (1999), respond to the deficiencies of the decision process methods.

In this sense, Kangas *et al.* (2001), Figueira *et al.* (2005) (following the studies of Roy, 1991; Roy and Bouyssou, 1993; and Schärlig, 1985), Tervonen, Figueira, Lahdelma and Salminen (2005), Hanandeh and El-Zein (2006), Wang (2007), and Afshari *et al.* (2010), among many many others, pointed out the relevance of multi criteria decision models, in particular, ELECTRE methods. So, ELECTRE methods are developed in two main phases. Firstly the *construction of the outranking relations*, and secondly the *exploitation of those relations* to get the final ranking of the alternatives. In the *exploitation procedure*, recommendations are elaborate from the results obtained in the first phase. The nature of the recommendation depends on the problematic: choosing, ranking or sorting. Each method is characterized by its construction and exploitation procedure.

Furthermore, these authors clarify that different ELECTRE methods may differ in how the outranking relations between the alternatives is done, and how they apply these relations to get the final ranking of the alternatives may differ.

Being ELECTRE method based on criteria's, it's important to distinct two sets of parameters: the *importance coefficients* and the *veto thresholds*. The *importance coefficients* in ELECTRE methods refer to intrinsic "weights". For a given criterion the weight, w_j, reflects its voting power when it contributes to the majority which is in favor of an outranking. The weights do not depend neither on the ranges nor the encoding of the scales. These parameters cannot be interpreted as substitution rates. The *veto thresholds* express the power attributed to a given criterion to be against the assertion "*a* outranks *b*", when the difference of the evaluation between *g*(*b*) and *g*(*a*) is greater than this threshold. These thresholds can be constant along a scale or it can also vary[6].

Briefly, ELECTRE approach considers thresholds and outranking. So, it is assumed a defined <u>criteria</u> g_j, j = 1, 2, …, r and a set of <u>alternatives</u> ***A*** (Buchanan *et al.*, 1999). If *in traditional modeling*, there are two relations for two alternatives (a, b_h) Є A, such that:

a*P*b$_h$ (a is preferred to b_h) <=> $g(a) > g(b_h)$

a*I*b$_h$ (a is indifferent to b_h) <=> $g(a) = g(b_h)$

in ELECTRE methods, an indifference threshold ***q***, a preference threshold ***p***, and an additional binary relation ***Q*** are introduced. So the above relations are redefined to:

a*P*b$_h$ (*a* is strongly preferred to b_h) <=> $g(a) - g(b_h) > p$

a*Q*b$_h$ (*a* is weakly preferred to b_h) <=> $q < g(a) - g(b_h) <= p$

a*I*b$_h$ (*a* is indifferent to b_h, and b to *a*) <=> $|g(a) - g(b_h)| <= q$

The definition of these thresholds will permit to outrank a relation a***S***b_h, this is, the idea is to test all the alternatives "*a* is at least as good as b_h" or "*a* is not worse than b_h", and validate, or no, the assertion a***S***b_h. So, this gives rise to one of the following four situations:

- [a***S***b_h and not(b_h***S***a)] ⇔ a***P***b (*a* is strictly preferred to *b*);
- [not(a***S***b_h) and b_h***S***a] ⇔ a***R***b (*a* is incomparable to *b*);
- [a***S***b_h and b_h***S***a] ⇔ a***I***b (*a* is indifferent to *b*);
- [not(a***S***b_h) and not(b_h***S***a)] ⇔ a***R***b (*a* is incomparable to *b*).

6 About this topic see, T. Saaty (1980), C. Bana e Costa and J. Vansnick (1994), R. Keeney and H. Raiffa (1976), J. Figueira and B. Roy (2002), L. Maystre, J. Pictet, and J. Simos (1994), V. Mousseau (1993), M. Rogers and M. Bruen (1998), M. Rogers, M. Bruen (2000), B. Roy and V. Mousseau (1996), B. Roy, M. Pr´esent, and D. Silhol (1986), J. Simos (1990), J. Vansnick (1986).

To test the assertion a***S***b_h (or b_h***S***a), two conditions should be verified:

- *Concordance condition*: for an outranking a***S***b_h (or b_h***S***a) to be accepted, a "sufficient" majority of criteria should be in favor of this assertion;

- *Non-Discordance condition*: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion a***S***b_h (or b_h***S***a) in a "too strong way".

As already mentioned, two types of inter-criteria preference parameters intervene in the construction of ***S***:

- The set of *weight-importance* coefficients (k_m, k = 1, 2, …, *m*) is used in the concordance test, when computing the relative importance of the coalitions of criteria being in favor of the assertion a***S***b_h, and
- The set of *veto thresholds* ($v_1(b_h)$, $v_2(b_h)$, …, $v_m(b_h)$), *h* Є B, is used in the discordance test v_j (b_h) represents the smallest difference $g_j(b_h) – g_j(a)$ incompatible with assertion a***S***b_h.

Finally, to enable the comparison on an alternative *a* to an attribute b_n, the relation is build through the following steps:

- Compute the partial concordance indices $c_j(a, b_h)$ and $c_j\ (b_h, a)$;
- Compute the overall concordance indices $c(a, b_h)$;
- Compute the partial discordance indices $d_j(a, b_h)$ and $d_j(b_h, a)$;
- Compute the fuzzy outranking relation grounded on the credibility indices $\delta(a, b_h)$;
- Determine a *λ-cut* of the fuzzy relation in order to obtain a crisp outranking relation.

As stated by Figueira *et al.* (2005), Tervonen *et al.* (2005), and Hanandeh and El-Zhein (2006), ELECTRE methods cannot be used for decision process without some external method, needed to transform the preferences into deterministic weight values. Although the innumerous weight elicitation techniques proposed (Mousseau, 1995; Hokkanen and Salminen, 1997; and Figueira and Roy, 2002), Rogers and Bruen (1998b) criticized the methods available for eliciting weighting values with ELECTRE III. They highlighted the fact that due to the non-compensatory nature of ELECTRE III, using weight averages does not give a true representation of the stakeholders' preferences. So, these authors approach uses pair wise comparisons to elicit the weights. They add that using weight elicitation techniques, the stability should be analyzed by using intervals for the weights, because the difficulty of expressing beliefs in mathematical terms causes inaccuracy in the evaluations.

Hereafter this general knowledgements on ELECTRE methodology, it is now appropriate to present the specific features of each version.

In this sense, Kangas *et al.* (2001), José Figueira *et al.* (2005), Tervonen *et al.* (2005), Huang and Chen (2005), Hanandeh and El-Zein (2006), Wang (2007), and Afshari *et al.* (2010) studies, guide us for each version, depending on the intended study: for *choice problem*, we can apply ELECTRE I, ELECTRE Iv, and ELECTRE IS; for *ranking problem*, we can apply ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE-SS; and for *sorting problem* we can apply ELECTRE TRI.

1. CHOICE PROBLEMATIC:

A decision maker under choosing problematic must be helped in selecting a subset of actions, as small as possible, in such a way that a single action may finally be chosen.

1.1. ELECTRE I (electre one):

Figueira *et al.* (2005) consider that this method does not have a significant practical interest, given the diversity nature of real world applications, which usually have a vast spectrum of quantitative and qualitative elementary consequences. This leads to the construction of a contradictory and very heterogeneous set of criteria, with both numerical and ordinal scales associated with them. In addition, a certain degree of imprecision or uncertainty is always attached to the knowledge collected from real-world problems.

The method is very simple and it should be applied only when all the criteria have been coded in numerical scales with identical ranges. In such a situation, the assertion $a\boldsymbol{S}b_n$ is valid, only when two conditions hold: the strength of the concordant condition must be powerful, and no discordance against the assertion "a is at least as good as b_n" may occur.

The first condition, the strength of the *concordant condition*, must be understood as the sum of the weights associated to the criteria forming that condition. It can be defined by the following *concordance index*:

$$c(a, b) = \sum_{j:gj(a)\geq gj(b)} wj \qquad [1]$$

where,

$\sum_{j \in J} w_j = 1$, where J is the set of the indices of the criteria;

$j{:}g_j(a) \geq g_j(b)$, is the set of indices for all the criteria belonging to the concordant condition with the outranking relation *aSb*.

In other words, the value of the concordance index must be greater than or equal to a given concordance level, *s*, whose value generally falls when c(*a,b*) ≥ s.

The second, and last condition, *no discordance* against the assertion "*a* is at least as good as *b*" may occur, is based on discordance measurement. The discordance is measured by a *discordance level* defined as follows:

$$d(a, b) = \max_{j:gj(a)<gj(b)} \left\{ g_j(b) - g_j(a) \right\} \qquad [2]$$

The power of the discordant condition tells us that, if its value surpasses a given level, *v*, the assertion is no longer valid. So, discordant condition exerts no power if d(*a, b*) ≤ v.

Both concordance and discordance indices have to be computed for every pair of actions *(a, b)* in the set *A*, where $a \neq b$. As already said, this computer procedure leads to a binary relation, where for each pair of action *(a, b)*, only one of the following situations may occur:

- a***S***b and not b***S***a ⇔ a***P***b (*a* is strictly preferred to *b*);
- b***S***a and not a***S***b ⇔ b***P***a (*b* is strictly preferred to *a*);
- a***S***b and b***S***a ⇔ a***I***b (*a* is indifferent to *b*);
- Not a***S***b and not b***S***a ⇔ a***R***b (*a* is incomparable to *b*).

One of the big disadvantages of ELECTRE I, is that this framework says nothing to decision maker about how to select the best compromise action, or a subset of actions. In the construction procedure (the first procedure) of ELECTRE I method only one outranking relation *S* is matter of fact. When exploiting this outranking relation (the second procedure) in order to identify a small as possible subset of actions, from which the best action could be selected, all the actions which form a cycle are considered indifferent. Because of this, ELECTRE I is criticized, giving place to ELECTRE IS, which was developed to mitigate this inconvenient.

1.2. ELECTRE Iv (electre one vee):

Continuing Figueira *et al.* (2005) study, ELECTRE Iv, is nothing more nothing less than, ELECTRE I with veto threshold (Maystre, Pictet, and Simos, 1994). The introduction of veto threshold, v_j, made possible for analysts and decision makers, to overcome the difficulties related to the heterogeneity of scales: whichever the scales type are, this method is always able to select the best compromise action or a subset of actions to be analyzed by decision makers.

In short, the concept of *veto threshold* is related to the definition of an upper bound beyond, which the discordance about the assertion "*a* outranks *b*" cannot surpass, allowing an outranking. Differently from ELECTRE I, where discordance level is related to the scale of criterion g_j in absolute terms for an action *a* from *A*, in ELECTRE Iv veto threshold is related to the preference differences between $g_j(a)$ and $g_j(b)$.

The mathematic formulation little differs from ELECTRE I, in the sense that the *discordance condition* is now called *no veto condition*, which may be stated as follows:

$$g_j(a) + v_j(g_j(a)) \geq g_j(b), \ \forall_j \in J \qquad [3]$$

Finally, to validate the assertion "*a outranks b*" it is necessary that, among the minority of criteria that are opposed to this assertion, none of them puts its veto.

Despite these improvements, the problem of imperfect knowledge remains.

1.3. ELECTRE IS (electre one esse):

According to Figueira *et al.* (2005), the main innovation of ELECTRE IS is the use of *pseudo-criteria* instead of *true-criteria*. This method takes into account, the possibility to use indifference and preference thresholds for certain criteria belonging to *F* and, correlatively, a backing up (reinforcement) of the veto effect when the importance of the concordant condition decreases. In the construction procedure, each condition is considered individually:

• *Concordance condition*:

- Condition of criteria in which ***aSb***:

$$J^S = \{ j \in J: g_j(a) + q_j(g_j(a)) \geq g_j(b) \} \quad [4]$$

- Condition of criteria in which ***bQa***:

$$J^Q = \{ j \in J: g_j(a) + q_j(g_j(a)) < g_j(b) \leq g_j(b) + p_j(g_j(b)) \} \quad [5]$$

So, concordance condition will be:

$$c(a, b) = \sum_{j \in JS} wj + \sum_{j \in JQ} \partial j Wj \geq S \quad [6]$$

where,

$$\varphi j = \frac{gj(a) + pj(gj(a)) - gj(b)}{pj(gj(a)) - qj(gj(a))}$$

(the coefficient φ_j decreases linearly from *1* to *0*, when g_j describes the range $[g_j(a) + q_j(g_j(a)), g_j(a) + p_j(g_j(a))]$.

• *No veto condition:*

$$g_j(a) + v_j(g_j(a)) \geq g_j(b) + q_j(g_j(b)) \; \eta_j \quad [7]$$

where,

$$\eta_j = \frac{1 - c(aSb) - wj}{1 - s - wj} \qquad [8]$$

In the exploitation procedure, actions belonging to a cycle are no longer considered as indifferent as in the previous versions of ELECTRE for choice problems.

2. RANKING PROBLEMATIC:

In ranking problematic, the question lays in the way to rank of all the actions belonging to a given set of actions, from the best to the worst. There are four different ELECTRE methods to deal with this problematic: ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE-SS.

Wang (2007) defends that there is a visible difference between ELECTRE II and ELECTRE III methods, this is, the use of different types of criteria. On one hand, ELECTRE II uses the *true criteria* where no thresholds exist and the differences between criteria scores are used to determine which alternative is preferred (the indifference relation is transitive (Rogers, *et al.*, 1999)). On the other hand, the criteria used by ELECTRE III are *pseudo criteria* which involve the use of two-tiered thresholds: the indifference threshold *q*, below which the decision maker shows clear indifference; and the preference threshold *p*, above which the decision maker is certain of strict preference (Rogers, *et al.*, 1999). The situation between the above two is regarded as weak preference for alternative *a* over alternative *b* which indicates the decision maker's hesitation between indifference and strict preference (Rogers, *et al.*, 1999).

2.1. ELECTRE II (electre two):

Concerning the ranking problem, ELECTRE II, was the first of ELECTRE methods especially designed to deal with this problems. Besides that, it is also important to point out that ELECTRE II, as stated by Figueira *et al.* (2005), was also the first method, to use a technique based on the construction of an embedded outranking relations sequence (*a strong outranking relation* followed by a *weak outranking relation*).

Figueira *et al.* (2005), Huang and Chen (2005), Wang (2007) and Wang and Triantaphyllow (2008) clarifies that, the construction procedure is much closer to ELECTRE Iv, in the sense that it is also a *true-criteria* procedure.

The ELECTRE methods are based on the evaluation of two indices, the *concordance* index and the *discordance* index, defined for each pair of alternatives. The *concordance index* for a pair of alternatives *a* and *b* measures the strength of the hypothesis that alternative "*a* is at least as good as alternative *b*" - $a\boldsymbol{S}b_h$. The *discordance index* measures the strength of evidence against this hypothesis (Belton and Stewart, 2001). There are no unique measures of concordance and discordance indices. In ELECTRE II, the *concordance index c(a, b)* for each pair of alternatives *(a, b)* is defined as follows:

$$c(a,b) = (\sum_{j \in Q(a,b)} w_j) / (\sum^{m}_{j=1} w_j) \qquad [8]$$

where,

Q (a, b) is the set of criteria for which *a* is equal or preferred to *b* (*as good as*), and

$\boldsymbol{w_j}$ is the weight of the *j-th* criterion.

And the *discordance index d (a, b)* for each pair of alternatives *(a, b)* is defined as follows:

$$d(a, b) = (\max_j (g_j(b) - (g_j(a)) / \delta \qquad [9]$$

where,

$\boldsymbol{g_j}$ ***(a)*** represents the performance of alternative *a* in terms of criterion c_j,

$\boldsymbol{g_j}$ ***(b)*** represents the performance of alternative *b* in terms of criterion c_j, and

$\boldsymbol{\delta}$ = max | g_j (*b*) – g_j (*a*) |, this is, the maximum difference on any criterion. This definition can only be used when the scores for different criteria are comparable.

After computing the concordance and discordance indices for each pair of alternatives, two types of outranking relations are built by comparing these indices with two pairs of threshold values: (c^*, d^*) and (c',d'). The pair (c^*, d^*) is defined as the concordance and discordance thresholds for the *strong* outranking relation, and the pair (c', d') is defined as the thresholds for the *weak* outranking relation where $c^* > c'$ and $d^* < d'$. The outranking relations are built according to the following two rules:

(1) If $c(a, b) \geq c^*$, $d(a, b) \leq d^*$ and $c(a, b) \geq c(b, a)$, then alternative *a* is regarded as strongly outranking alternative *b*.

(2) If $c(a, b) \geq c'$, $d(a, b) \leq d'$ and $c(a, b) \geq c(b, a)$, then alternative *a* is regarded as weakly outranking alternative *b*.

The values of *(c*, d*)* and *(c', d')* are decided by the decision maker for a particular outranking relation: the higher the value of *c** and the lower the value of *d**, the more severe the outranking relation becomes, that is, the more difficult it is for one alternative to outrank another (Belton and Stewart, 2001).

To determine outranking relations, descending and ascending distillation processes are applied to obtain two complete pre-orders of the alternatives, (Belton and Stewart, 2001; and Rogers *et al.*, 1999). The *descending pre-order* is built up by starting with the set of "best" alternatives (those which outrank other alternatives) and going downward to the worse one. On the contrary, the *ascending pre-order* is built up by starting with the set of "worst" alternatives (those which are outranked by other alternatives) and going upward to the best one.

The last step is to combine the two complete pre-orders to get either a *partial or a complete final pre-order*. Having a *partial pre-order* (not containing a relative ranking of all the alternatives) or a *complete pre-order*, depends on the level of consistency between the rankings from the two distillation procedures (Rogers *et al.*, 1999). The *partial pre-order* allows two alternatives to remain incomparable without affecting the validity of the overall ranking, which differentiates from the *complete pre-order*. A commonly used method for determining the *final pre-order* is to take the intersection of the *descending and ascending pre-orders*. The intersection of the two pre-orders is defined such that alternative *a* outranks alternative *b* (***aSb***) if and only if *a* outranks or is in the same class as *b* according to the two pre-orders. If alternative *a* is preferred to alternative *b* in one pre-order but *b* is preferred to *a* in the other one, then the two alternatives are incomparable in the final pre-order (Rogers *et al.*, 1999).

The main problem with this method, as stated by Huang and Chen (2005) and Wang (2007) is the occurrence of rank reversals[7]. They add that, the main reason for rank reversals lies in the exploitation of the pair wise outranking relations, that is, the upward and downward distillation processes. The basic idea behind the distillation processes is to decide the rank of each alternative by the degree of how this alternative outranks all the other alternatives. When a non-optimal alternative in an alternative set is replaced by a worse one, the pair wise outranking relations related to it may be changed accordingly and the overall ranking of the whole alternative set, which depends on those pair wise outranking relations, may also be changed. The first change is reasonable when considering the fact that a non-optimal alternative has been replaced by a worse one. However, the second change is unreasonable and may cause undesirable rank reversals.

[7] Reliability and validity of ELECTRE methods is detailed on Chapter V.

2.2. ELECTRE III (electre three):

As stated by Buchanan *et al.* (1999), Figueira *et al.* (2005), Tervonen *et al.* (2005), and Serdar and Ayner (2009), ELECTRE III (from Roy, 1978), being the mostly used method, is a well-established multi criteria decision maker method that has a history of successful in real-life (see also Georgopoulou *et al.*, 1997; Hokkanen and Salminen, 1997; Karagiannidis and Moussiopoulos, 1997; and Rogers *et al.*, 1999 among many others).

In ELECTRE III the outranking relation can be interpreted as a fuzzy relation. The construction of this relation requires the definition of a *credibility index* (which characterizes the credibility of the assertion $a\boldsymbol{S}b_h$ - "*a outranks b*" – being defined by using the *concordance index* and a *discordance index* for each criterion g_j in F.

The concordance index $c_j(a, b)$ calculated for each pair of alternatives (a, b) in terms of each one of the decision criteria, follows the formula:

$$c_j\,(a, b) = \begin{cases} 1 & \text{if } g_j(a) + q_j\,(g_j(a)) \geq g_j(b) \\ 0 & \text{if } g_j(a) + p_j\,(g_j(a)) \leq g_j(b) \\ g_j(a) + q_j\,(g_j(a)) < g_j(b) < g_j(a) + p_j\,(g_j(a)), & \text{otherwise} \end{cases} \quad [10]$$

were, $q_j\,(.)$ and $p_j\,(.)$ are the indifference and preference threshold values for criterion c_j (Belton and Stewart, 2001).

The next step is to calculate the discordance index $d_j\,(a, b)$ for all the alternatives in terms of each one of the decision criteria according to the following formula:

$$d_j(a, b) = \begin{cases} 1 & \text{if } g_j\,(b) \geq g_j(a)+v_j(g_j(a)) \\ 0 & \text{if } g_j\,(b) \leq g_j(a)+p_j(g_j(a)) \\ g_j(a)+p_j(g_j(a)) < g_j\,(b) < g_j(a)+v_j(g_j(a)), & \text{otherwise} \end{cases} \quad [11]$$

where, $v_j\,(.)$ is the veto threshold for criterion c_j (Belton and Stewart, 2001). If no veto threshold is specified, then $d_j(a, b) = 0$ for all pairs of alternatives.

Finally, the credibility index $\rho(a, b)$ is defined as follows,

$$\rho(a, b) = \begin{cases} c(a, b), & \text{if } d_j(a, b) \leq c(a, b),\ j = 1, \ldots, n \\ c(a, b) \prod_{j \in J(a,b)} \dfrac{1 - dj(a,b)}{1 - cj(a,b)}, & \text{otherwise} \end{cases} \qquad [12]$$

where,

$\boldsymbol{c(a, b)} = (\sum^{m}_{j=1} w_j\, c_j(a, b)) / (\sum^{m}_{j=1} w_j)$

J (a, b) is the set of criteria for which $d_j(a, b) > c(a, b)$. The credibility index is a measure of the strength of the claim that "alternative *a* is at least as good as alternative *b*" - ***aSb***.

To notice that, when $d_j(a,b) = 1$, it implies that $\rho(a, b) = 0$, since $c(a, b) < 1$.

Next, the descending and ascending distillations procedures (Belton and Stewart, 2001 and Rogers *et al.*, 1999) must be applied based on the credibility index, in order to construct the two pre-orders for the alternatives. Being defined the two pre-orders, they are combined to get the final overall ranking of the alternatives. The way to combine the two pre-orders follows ELECTRE II procedure.

As already point out to ELECTRE II method, the same criticism could be applied to ELECTRE III method, this is, the occurrence of rank reversals, as stated by Kangas *et al.* (2001), Tervonen *et al.* (2005), and Wang (2007), among others[8].

2.3. ELECTRE IV (electre four):

Figueira *et al.* (2005) clarifies ELECTRE IV is also a procedure based on the construction of a set of embedded outranking relations. There are five different relations, $S^1, \ldots, S^5$. The S^{r+1} relation (r = 1, 2, 3, 4) accepts an outranking in a less credible circumstances than the relation S^r. It means (while remaining on a merely ordinal basis) the assignment of a value ρ_r for the credibility index ρ(a, b) to the assertion ***aSb***. The chosen values must be such that $\rho_r > \rho_{r+1}$. Furthermore, the movement from one credibility value ρ_r to another ρ_{r+1} must be perceived as a considerable loss.

The ELECTRE IV exploiting procedure is the same as in ELECTRE III.

[8] Reliability and validity of ELECTRE methods is detailed on Chapter V.

2.4. ELECTRE-SS (electre stochastic):

Hanandeh and El-Zhein (2006) proposed a modified version of ELECTRE III, called ELECTRE-SS, which uses stochastic techniques to account for uncertainty in the weightings and threshold values of criteria. This method is particularly useful when, in particular, a large number of decision makers are involved in the decision-making process.

In ELECTRE III, both thresholds, *p* and *q*, are treated as fixed values, and criteria weights are deterministic values. However, this involves not only the error estimation in each criterion, but also subjective input of the decision maker (Rogers and Bruen, 1998a).

In order to overcome this flaw, Hanandeh and El-Zhein (2006) introduced a new stochastic method to allow for multiple decision makers input through accepting criteria weights and thresholds as ranges, rather than deterministic values.

As ELECTRE III method, ELECTRE-SS follows similar procedures: *outranking phase* and *exploitation phase*. The outranking phase builds an outranking index by forming an outranking relation between the pairs of alternatives. The outranking index is then exploited in the second phase to produce a partial pre-order.

Considering mathematical formulation, indifference threshold q'_j and preference threshold p'_j , are defined as stochastic variables, instead of being deterministic values, and can vary along the scale of the criteria value. Hence, Hanandeh and El-Zhein (2006) rewrite the preference and indifference relations as follows:

- $aSb^j \Leftrightarrow g_j(a) > g_j(b) + q'_j$ [13]
- $aIb^j \Leftrightarrow |g_j(a) - g_j(b)| \leq q'_j$

- $aP'b^j \Leftrightarrow g_j(a) > g_j(b) + p'_j$
- $aQ'b^j \Leftrightarrow q'_j < g_j(a) - g_j(b) \leq p'_j$ [14]
- $aI'b^j \Leftrightarrow |g_j(a) - g_j(b)| \leq q'_j$

This way, both values of *w* (criteria importance index – weight) and threshold values (*p* and *q*) are defined to accommodate decision maker evaluations and level of confidence in their evaluations. When the number of decision makers is significantly large, a probability distribution function can be built to represent the entire spectrum of evaluations. However, when the number of decision makers is not large enough to derive a probability distribution function, then the lowest value and the highest value are taken and a normal distribution is considered for evaluating the results in between the min-max range. Hence,

$$\begin{aligned}
&- w'_j = w^{min}_j + (w^{max}_j - w^{min}_j) \text{ x } \Delta^w_j \\
&- q'_j = q^{min}_j + (q^{max}_j - q^{min}_j) \text{ x } \Delta^q_j \\
&- p'_j = p^{min}_j + (p^{max}_j - p^{min}_j) \text{ x } \Delta^p_j
\end{aligned} \qquad [15]$$

were,

Δ is the probability distribution function fit for the importance index of criteria *j*.

$\mathbf{\Delta^q_j}$ and $\mathbf{\Delta^p_j}$ are the probability distribution functions of the indifference and preference thresholds for criteria *j* respectively.

The *credibility index ρ(a,b)* for the outranking relation *aSb*, is defined using both a comprehensive *concordance index c(a,b)* and a discordance index $d_j(a, b)$ for each criterion $g_j \in G$. As ELECTRE III, partial concordance index can be defined as,

$$c'_j(a, b) = \begin{cases} 1, & g_j(a) + q'_j \geq g_j(b) \\ 0, & g_j(a) + p'_j \leq g_j(b), \text{ where } j=1, \ldots, n \\ [p'_j + g_j(a) - g_j(b)] / [(p'_j - q'_j)], & \text{otherwise} \end{cases} \qquad [16]$$

Since Hanandeh and El-Zhein (2006) do not consider a veto threshold, the discordance index is zero for all criteria. Therefore, the credibility index *ρ' (a, b)* in this case is equal to the comprehensive concordance index *c' (a, b)*. So, the comprehensive concordance index is then calculated as follows:

$$c'(a, b) = (1/K') \sum_{j=1}^{n} w'_j \text{ x } c'_j(a, b), \text{ where } K' = \sum_{j=1}^{n} w'_j \qquad [17]$$

To exploit the outranking matrix, two complete pre-orders are constructed:

- Z'_1, a descending distillation: $Z'_1 = \{z'_{1,1}, z'_{1,l}, \ldots, z'_{1,k}\}$
- Z'_2, an ascending distillation: $Z'_2 = \{z'_{2,1}, z'_{2,l}, \ldots, z'_{2,k}\}$

where,

$z'_{1,l}$, $z'_{2,l}$ are the number of times alternative a_i ranked in the k^{th} order in the descending and ascending distillations respectively.

Then, two complete pre-orders Z_1, Z_2 were built such that,

$$Z_1 = z'_{1,1} + \sum_{l=1}^{k} - l \text{ x } z'_{1,l}$$

$$Z_2 = z'_{2,1} + \sum_{l=1}^{k} - l \text{ x } z'_{2,l}$$

Finally a partial order is constructed as follows,

$$Z = Z_1 \wedge Z_2$$

To test ELECTRE-SS method, Hanandeh and El-Zhein (2006) study a case already published (see Rogers *et al.*, 1999). This case is about the definition of an "optimum waste strategy for the region", requested by the Federal Agency for the Environment in Switzerland. Evaluating municipal solid waste management alternatives usually involves a great deal of uncertainty, especially when considering social and environmental criteria. The region was divided into four zones for planning purposes, and eleven strategic options were identified for further assessment against eleven environmental, economic, political and technical criteria (alternative A_{ij} was evaluated in terms of criteria C_k, were i=1, …, 4, j=1, …3 and k=1, … 4). Beyond that, four major criteria categories were considered in the decision making: Environmental criteria (C1), Economic (C2), Technical (C3) and Political (C4). Each criterion is further divided into sub-criteria.

To apply ELECTRE-SS, the values of p and q are not known, but fall into a range defined by the authors, and each weights falls between the lowest and largest assigned values for each criteria used as in the original case. Running ELECTRE-SS method, Hanandeh and El-Zhein (2006) conclude that final ranking of alternatives is sensitive to both threshold and criteria weight values, and the final ranking is more sensitive to criteria weights than threshold values. Besides this, they find that average weights are not necessarily good estimates of criterion weights.

Hanandeh and El-Zhein (2006) reinforced that the new method ELECTRE-SS has the advantage of assessing the performance reliability of the selected alternative, which is not possible when using the deterministic ELECTRE III method. It also allows for close analyze of each alternative's performance, hence decisions may include alternatives that otherwise may be excluded if deterministic parameters where used. Finally, the method provides easy presentation of results in tabular format that gives the decision maker a clear ranking which can be further inspected using the graphical presentation mode.

3. SORTING PROBLEM:

In sorting problematic, each action is considered independently from the others in order to determine the categories to which it seems justified to assign it, by means of comparisons to profiles (bounds, limits), norms or references. Results are expressed using the absolute notion of "assigned" or "not assigned" to a category, "similar" or "not similar" to a reference profile, "adequate" or "not adequate" to some norms. The sorting problematic refers thus to absolute judgements.

3.1. ELECTRE TRI (electre tree):

ELECTRE TRI, also a very well-successful model in real life, is designed to assign a set of actions, objects or items to categories. In ELECTRE TRI categories are ordered from the worst (C_1) to the best (C_k) (see Dias *et al.*, 2002; Damart *et al.*, 2007; Xidonas *et al.*, 2009; Coelho *et al.*, 2009; Bregar *et al.*, 2009; and Sobral, 2010 among others) .

Each category must be characterized by a lower and an upper profile, were $C = \{C_1, . . . ,C_h, . . . ,C_k\}$ denote the set of categories. The assignment of a given action a to a certain category C_h results from the comparison of a to the profiles defining the lower and upper limits of the categories: being b_h the upper limit of category C_h, and the lower limit of category C_{h+1}, for all $h = 1, . . . , k$. For a given category limit, b_h, this comparison rely on the credibility of the assertions $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$. This credibility (index) is defined as in ELECTRE III.

After determining the credibility index, a λ - *cut* level of the fuzzy relation must be introduced in order to obtain a crisp outranking relation. This level can be defined as the credibility index smallest value, compatible with the assertion $a\boldsymbol{S}b_h$.

Being P the preference, I the indifference relation and R the incomparability binary relations, action a and profile b_h may be related to each other as follows:

a) aIb_h if $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$

b) aPb_h if $a\boldsymbol{S}b_h$ and not $b_h\boldsymbol{S}a$

c) b_hPa if not $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$

d) aRb_h if not $a\boldsymbol{S}b_h$ and not $b_h\boldsymbol{S}a$

The objective of the exploitation procedure is to exploit the above binary relations, and propose an assignment, in particular,

1. The *conjunctive logic*, in which an action can be assigned to a category when its evaluation on each criterion is at least as good as the lower limit which has been defined on the criterion to be in this category. The action is hence assigned to the highest category fulfilling this condition.

2. The *disjunctive logic*, in which an action can be assigned to a category, if it has, on at least one criterion, an evaluation at least as good as the lower limit which has been defined on the criterion to be in this category. The action is hence assigned to the highest category fulfilling this condition.

With *disjunctive rule*, the assignment of an action is generally higher than with the *conjunctive rule*. This is why the *conjunctive rule* is usually interpreted as pessimistic while the disjunctive rule is interpreted as optimistic. This interpretation (optimistic-pessimistic) can be permuted according to the semantic attached to the outranking relation.

When no incomparability occurs in the comparison of an action *a* to the limits of categories, *a* is assigned to the same category by both the optimistic and the pessimistic procedures. When *a* is assigned to different categories by the optimistic and pessimistic rules, *a* is incomparable to all "intermediate" limits within the highest and lowest assignment categories. ELECTRE TRI is a generalization of the two above mentioned rules. The two procedures can be stated as follows,

1. *Pessimistic rule:* An action *a* will be assigned to the highest category C_h such that $a\boldsymbol{S}b_{h-1}$.

a) Compare *a* successively with b_h, h = k − 1, k − 2, . . . , 0.

b) The limit b_h is the first encountered profile such that $a\boldsymbol{S}b_h$.

Assign *a* to category C_{h+1}.

2. *Optimistic rule:* An action *a* will be assigned to the lowest category C_h such that $b_h\boldsymbol{P}a$.

a) Compare *a* successively with b_h, h = 1, 2, . . . , k − 1.

b) The limit b_h is the first encountered profile such that $b_h\boldsymbol{P}a$.

Assign *a* to category C_h.

IV – SOME TEST CRITERIA FOR EVALUATING THE MULTI CRITERIA DECISION MAKING METHODS

In Triantaphyllou (2000), Wang and Triantaphyllou (2004, 2008), and Wang (2007) studies, three test criteria were established to evaluate the performance of multi criteria decision making methods by testing the validity of their ranking results. These test criteria are as follows:

Test Criterion #1: "*An effective multi criteria decision making method should not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative (given that the relative importance of each decision criterion remains unchanged)."*

Suppose that a multi criteria decision making methods has ranked a set of alternatives in some way. Next, suppose that a non-optimal alternative, say *Ak*, is replaced by another alternative, say *Ak'*, which is less desirable than *Ak.* Then, according to test criterion #1, the indication of the best alternative should not change when the alternatives are ranked again by the same method. The same should also be true for the relative rankings of the rest of the unchanged alternatives.

Test Criterion #2: *"The rankings of alternatives by an effective multi criteria decision maker method should follow the transitivity property."*

Suppose that a multi criteria decision maker method has ranked a set of alternatives of a decision problem in some way. Next, suppose that this problem is decomposed into a set of smaller problems, each defined on two alternatives at a time and the same number of criteria as in the original problem. Then, according to this test criterion all the rankings which are derived from the smaller problems should satisfy the transitivity property. That is, if alternative *A1* is better than alternative *A2*, and alternative *A2* is better than alternative *A3*, then one should also expect that alternative *A1* is better than alternative *A3*.

Test Criterion #3: *"For the same decision problem and when using the same multi criteria decision maker method, after combining the rankings of the smaller problems that an multi criteria decision maker problem is decomposed into, the new overall ranking of the alternatives should be identical to the original overall ranking of the undecomposed problem."*

As before, suppose that a multi criteria decision maker problem is decomposed into a set of smaller problems, each defined on two alternatives and the original decision criteria. Next suppose that the rankings of the smaller problems follow the transitivity property. Then, according to this test criterion when the rankings of the smaller problems are all combined together, the new overall ranking of the alternatives should be identical to the original overall ranking before the problem decomposition.

In Triantaphyllou (2000), Wang and Triantaphyllou (2004, 2008), and Wang (2007) research, these three test criteria were used to evaluate the performance of the ELECTRE II and the ELECTRE III methods. Both of them failed in terms of each one of these three test criteria.

In the next chapter – Chapter V – a few examples are presented, developed by Kangas *et al.* (2001), Tervonen *et al.* (2005), Huang and Chen (2005), Wang and Triantaphyllou (2004, 2008) and Wang (2007), in order to demonstrate that rank reversal may occur with ELECTRE methodology, in particular, ELECTRE II and ELECTRE III.

V – RELIABILITY AND VALIDITY OF ELECTRE METHODS

In Kangas *et al.* (2001) study, ELECTRE III and PROMETHEE II methods were tested. Like others authors, they identified rank reversals in both methods. In any decision making process there is uncertainty concerning not only the values of the criterion variables but also concerning, for example, the weights of the criteria. So, a sensitivity analysis with respect to the uncertain parameters used in the calculations is thus essential, as well as the application of several alternative methods to the same problem. Then, the decision makers can make the final choice among these alternative solutions (Salminen *et al.,* 1998).

While enhancing outranking methods advantages[9], Kangas *et al.* (2001) study confirms that if the priority of one alternative depends on other alternatives, this means that adding a new (non-optimal) alternative, a change in ranks of the initial alternatives may occurs.

Tervonen *et al.* (2005) also reported problems when applying ELECTRE III: concerning preference information, if the decision makers cannot provide precise and complete weight information, or if there are multiple decision makers with conflicting preferences, ELECTRE methods cannot be used for decision process. To comprove this assertation, Tervonen *et al.* (2005) re-analyze the case study presented in Rogers *et al.* (1999): to choose the best waste incineration strategy for the Eastern Switzerland region, considering eleven alternative strategies S_i that were evaluated in terms of eleven criteria C_j.

To perform this analysis, Tervonen *et al.* (2005) introduce an inverse weight-space analysis into the ELECTRE III method to explore the weight space, in order to describe which weights (weight intervals) result in certain ranks for the actions, meaning that no deterministic weights are required. This will allow ELECTRE III to be used with weight information of arbitrary type. This inverse approach on ELECTRE III was motivated by:

1. this type of weight information can be provided by the existing weight elicitation techniques[10];

2. it allows a particular kind of easily comprehensible "robustness analysis" also in the case when the weights are deterministic, and

3. if there are multiple decision makers whose preferences need to be taken into account, the weight intervals can be determined to contain the preferences of all decision makers.

So, Tervonen *et al.* (2005) analyzed robustness with respect to the weights, but considered all the others parameters fixed (thresholds, cutting levels, among others). Tervonen *et al.* (2005) adds that, usually tests to comprove robustness of multi criteria decision method are based on sensitive analysis by changing only a discrete set of weights for a criterion, or by considering only the extremes of the feasible weight space (Dias *et al.*, 2002). In their work, they consider an inverse approach on

[9] Ability to deal with uncertain and fuzzy information, ability to deal with ordinal and other informal preference statements, and the preference estimation procedures are versatile and diverse.

ELECTRE III, performing an inverse weight-space analysis, to all possible weight vectors in the feasible weight space.

Executing Monte Carlo simulations, which provides sufficient accuracy for the results (according to Lahdelma *et al.*, 2004), Tervonen *et al.* (2005) by changing only a single weight at a time concluded that, on one hand, in the original case study, alternatives S3.1 and S4.1 shared the best rank (based on analyzing six different sets of weights) leading to recommend S4.1 as the primary choice, and S3.1 as the secondary choice. But based on inverse analysis, alternative S4.1 seemed not to be the most adequate to "recommend" as the most favorable option, given its rank acceptability index. With 99% of those weights, it shared the first rank with alternative S3.1. On the other hand, S3.1 obtained lower rank than S4.1 with only 1% of the feasible weights, and was always ranked higher than the other alternatives (excluding S4.1).

In short, Tervonen *et al.* (2005) over this example would not "recommend" S4.1, because even small variations in the weights drop it below S3.1 in the ranking. In this sense, they would have select S3.1, and S4.1 as a "back-up" strategy, if for some reason S3.1 could not have been chosen. Tervonen *et al.* (2005) adds that this same analysis could be done using PROMETHEE method (see Figueira *et al.*, 2005).

Huang and Chen (2005) evaluated ELECTRE II model performance, in their study about land redevelopment in Heping Harbor Zone**,** in Taiwan. The authors suggested that when evaluating ELECTRE method,

- the definition and calculation of benchmarks of concordance index and discordance index are important elements;
- the method can be applied in parallel with other evaluation methods owing to getting only partial ranking (advantages and disadvantages of the combination of various evaluation methods, as well as their differences, should be weighted);
- the definition of weighs to be considered in the model is determined beforehand. Seeing this have a great impact upon the final ranking, particular attention shall be placed on weighting methodology when using ELECTRE evaluation method.

Wang and Triantaphyllou (2004, 2008) and Wang (2007) developed a very interesting paper about rank reversals, when using ELECTRE II and ELECTRE III to rank a set of decision alternatives.

[10] See page 13 of this work.

So, to check on rank reversals using ELECTRE II, their test were based on a real-life case study, were the aim's study was to help find the best location for a wastewater treatment plant in Ireland (Rogers *et al.*, 1999).

To check on rank reversals on ELECTRE III, their test were also based on a real-life case study, were the aim's study were to choose the best waste incineration strategy for the eastern Switzerland region (Rogers, *et al.,* 1999).

Concerning the *first problem*, to help find the best location for a wastewater treatment plant in Ireland, Wang (2007), defined the decision problem using five alternatives and seven criteria (set as a benefit criteria, that is, the higher the score the better the performance is) - alternative A_i was evaluated in terms of criteria C_j.

So, applying ELECTRE II methodology Wang (2007), construct the following pre-orders. From the descending distillation, ***A2= A5> A3> A1> A4***; from the ascending distillation, ***A2>A5= A3> A1> A4***. From the intersection of the descending and ascending pre-orders, the following complete pre-order of the alternatives were obtained: ***A2>A5> A3> A1> A4*** and obviously ***A2*** is the optimal alternative at this point.

Based on this results, Wang (2007) randomly selected ***A3*** to be replaced by a worse one, ***A3'***, in order to test ranking alternative's under the first test criterion.

After applying ELECTRE II methodology, the descending and ascending distillation processes allowed to reach the following results: the descending pre-order now is ***A2= A5> A3= A1> A4***, while the ascending pre-order is ***A2=A5>A3=A1>A4***. After combining the two pre-orders together, a new complete pre-order is got as follows: ***A2=A5>A3= A1> A4***. Now the best ranked alternatives are ***A2*** and ***A5*** together, a contradiction from the previous result which had ***A2*** as the only optimal alternative.

Considering the second problem, to choose the best waste incineration strategy for the eastern Switzerland region, Wang (2007), defined the decision problem using eleven alternatives and eleven criteria (*benefit criteria* means that the higher the score of a given criterion is, the more preferable it is; the *cost criteria* means that the lower the score of a given criterion is, the more preferable it is) - alternative A_i was evaluated in terms of criteria C_j.

So, applying ELECTRE III methodology Wang (2007), the pre-order obtained from the descending distillation was ***A9> A4> A7> A10> A3= A5= A8= A11> A1> A2> A6***. The pre-order obtained from the ascending distillation was ***A1= A7> A9> A4> A10> A2= A5> A3= A11> A8> A6.*** Then the two pre-orders were combined to get the final overall ranking of the alternatives, just like in ELECTRE II. Wang (2007) concluded that ***A7*** and ***A9*** are both regarded as the best-ranked alternatives because both of them are ranked first in the final partial pre-order. As a result, the rest of the alternatives were regarded as non-optimal ones.

As done to the first example, Wang (2007), selected, randomly, alternative ***A1*** and replaced it by a worse one′, ***A1'***, to test the reliability of the alternatives' ranking. Applying ELECTRE III method, Wang (2007) get the descending pre-order as ***A7> A9> A4> A10> A3 = A5 = A8= A11> A1> A2> A6***, and the ascending pre-order as ***A7> A1=A9> A4> A10> A2= A5> A3= A11> A8> A6***.

With these new results, the author concluded that the best-ranked alternative is only ***A7*** which is different from the original conclusion, which had ***A7*** and ***A9*** as the best-ranked alternatives.

Wang (2007), after analyzing both test results, explain that the reason for the contradictory results lies in the exploitation of the pair wise outranking relations, that is, the upward and downward distillation processes of ELECTRE II and ELECTRE III.

The basic idea behind the distillation processes is to decide the rank of each alternative by the degree of how this alternative outranks all the other alternatives. When a non-optimal alternative in an alternative set is replaced by a worse one, the pair wise outranking relations related to it may be changed accordingly, and the overall ranking of the whole alternative set, which depends on those pair wise outranking relations, may also be changed. The first change is reasonable when considering the fact that a non-optimal alternative has been replaced by a worse one. However, the second change is unreasonable and may cause undesirable rank reversals as confirmed by the above examples.

In short, this rank reversal happens because there is not a priori ranking of the alternatives when they are ranked by the ELECTRE II or III methods; the ranking of an individual alternative derived by these methods depends on the performance of all the other alternatives currently under consideration. This causes the ranking of the alternatives to depend on each other. Thus, it is likely that the optimal alternative may be different and the ranking of the alternatives may be distorted to some extent if one of the non-optimal alternatives in the alternative set is replaced by a worse one.

In Wang (2007) study, another example is analyzed, considering three alternatives ***A1***, ***A2***, and ***A3***. So, supposing originally that: ***A1*** strongly outranks ***A3***, ***A2*** weakly outranks ***A3,*** and ***A1*** and ***A2*** are indifferent with each other. The ranking of these three alternatives will be ***A1 > A2 > A3*** when using the ELECTRE II method.

Then, considering that the non-optimal alternative ***A3*** is replaced by a worse one. As a result of ranking process, ***A2*** may strongly outrank ***A3*** while ***A1*** is still strongly outranking ***A3,*** and ***A1*** is still indifferent with ***A2***. Now the ranking of the three alternatives will be ***A1= A2> A3*** by using the same method since both ***A1*** and ***A2*** now strongly outranks ***A3,*** and they are indifferent with each other. It can be seen that ***A1*** and ***A2*** are ranked equally now because ***A3*** becomes less desirable.

The situation above described is exactly what happened in the *first example*: ***A2*** and ***A5*** are ranked equally after ***A3*** has been replaced by a less desirable alternative. This kind of irregular situation is undesirable for a practical decision-making problem though it is reasonable in terms of the logic of the ELECTRE II method. It could leave the ranking of a set of alternatives to be manipulated to some extent.

Wang (2007) adds that, if the number of alternatives of a decision problem is more than 3, then the situation may become worse by totally changing the indication of the best ranked alternative. As pointed out by Belton and Stewart (2001), the results of the distillations are dependent on the whole alternative set, so that the addition or removal of an alternative may change some of the preferences between the remaining alternatives. That is, even without the addition or removal of alternatives, the best ranked alternative might be another one, and the previous pre-order between the remaining alternatives might be changed to some degree by just replacing a non-optimal alternative by a worse one.

Besides this distillation problem, Wang (2007) add that there is another factor that may contribute to rank reversals. During the construction of the pair wise outranking relations, both ELECTRE II and III need to use a value or a threshold which is also dependent on the performance values of all the currently considered alternatives. For ELECTRE II, it is the parameter d (the maximum difference of any criterion) in the discordance index formula. For ELECTRE III, it is the parameter used to decide the l (*preference* relations between the alternatives during the distillations). Both d and l values may be change when a non-optimal alternative is replaced by a worse one. Then the previous outranking relations between the other unchanged alternatives may be distorted to some degree, which finally may modify the indication of the best ranked alternative or the overall ranking of the alternatives.

In this sense, and based on the tests carried out, Wang (2007) concluded that the above two factors may function together or separately to cause rank reversals. So, this author inferred that ELECTRE II and III are not reliable and robust enough to offer a firm answer to a decision problem. In this sense, decision maker should undertake some kind of sensitivity and careful when analyzing the final rankings. Because of all this, when ranking alternatives, other methods can be considered, for example, AHP (from Saaty, 1980) or PROMETHEE family (from Brans and Vincke, 1985).

VII – CONCLUSION REMARKS

The aim of multi criteria decision models is to solve problems which require the inclusion of alternatives' judgments (choice alternatives) from various points of view (Escobar-Toledo and López-Garcia, 2005). These methods have been widely used in many real-life decision problems (Buchanan *et al.*, 1999; Zak, 2005; Kangas *et al.*, 2001; Huang and Chen, 2005; Ulubeyli and Kazaz, 2009; Afshari *et al.*, 2010, among many others). Therefore, these methods provide the methodological basis for solving the multifaceted problems and build realistic models and processes, for instance, in portfolio management they take into account, besides the two basic factors, risk and return (from the classic mean-variance model), one number of important additional factors such as market liquidity, PER, dividends growth rate, social responsibility, environmental protection, employee welfare, among others (Steuer and Na, 2003).

Determining decision criteria requires elaborating all properties of a desired post-implementation outcome. An analytic task, stated by the analyst and the decision maker, reflect particular aspects of considered implementation on characteristics describing possible options (decision alternatives). According to Roy (1991), there are three types of problems, *the choice problematic* (finding a subset of the set *A* which includes only the best solutions), *the sorting problematic* (assigning alternatives to defined categories), and *the ordering problematic* (constructing a ranking of alternatives in the set *A* from the best one to the worst one). Such an approach only considers a part of the decision process. Applying multi criteria methods to analyze a decision situation requires making a deliberate choice of a method suitable for a given decision situation. This decision process phase, the *exploitation phase*, intends to make a representation of the global preference which is the outcome of a decision maker's expectations and mutual local preferences between particular decision alternatives. Zbigniew and Watróbski (2008) indicate some methods suggested by the literature, for instance, TOPSIS (from Hwang and Yoon, 1981), AHP (from Saaty, 1980), ELECTRE family (from Roy, 1991), PROMETHEE family (from Brans and Vincke, 1985), ADELAIS (from Zopounidis *et al.* 1993), and MINORA (from Zopounidis, 1992).

Within all these models, we can highlight the ELECTRE family, from the "European school" which, as stated by Buchanan *et al.* (1999), respond to the deficiencies of the decision process methods.

Figueira *et al.* (2005) and Wang (2007), tells us that ELECTRE methods are developed into two main phases. Firstly the *construction of the outranking relations*, and secondly the *exploitation of those relations* to get the final ranking of the alternatives. In the *exploitation procedure*, recommendations are elaborate from the results obtained in the first phase. The nature of the recommendation depends on the problematic: choosing, ranking or sorting. Each method is characterized by its construction and exploitation procedure. Furthermore, these authors clarify that different ELECTRE methods may differ in how the outranking relations between the alternatives and how they apply these relations to get the final ranking of the alternatives may differ.

Being ELECTRE method based on criteria's, it is important to distinct two sets of parameters: the *importance coefficients* and the *veto thresholds*. The *importance coefficients* in ELECTRE methods refer to intrinsic "weights". For a given criterion the weight, w_j, reflects its voting power when it contributes to the majority which is in favor of an outranking. The weights do not depend neither on the ranges nor the encoding of the scales. These parameters cannot be interpreted as substitution rates. The *veto thresholds* express the power attributed to a given criterion to be against the assertion "a outranks b", when the difference of the evaluation between $g(b)$ and $g(a)$ is greater than this threshold. These thresholds can be constant along a scale or it can also vary. Thus, ELECTRE family deals with the three types of problems mentioned above: for *choice problem*, we can apply ELECTRE I, ELECTRE Iv, and ELECTRE IS; for *ranking problem*, we can apply ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE-SS; and for *sorting problem* we can apply ELECTRE TRI. These ELECTRE versions are discussed, for example, in Kangas *et al.* (2001), Figueira *et al.* (2005), Tervonen *et al.* (2005), Huang and Chen (2005), Hanandeh and El-Zein (2006), Wang (2007), and Afshari *et al.* (2010) studies.

But, since ELECTRE family, as any other model, is a representation of reality, it is possible to find theoretical fails, for instance, rank reversals. This fact was studied by a few authors. Kangas *et al.*(2001) tested ELECTRE III and PROMETHEE II methods. Their study revealed that if the priority of one alternative depends on other alternatives, so adding a new (non-optimal) alternative, a change in ranks of the initial alternatives may occur. Tervonen *et al.* (2005) also reported problems when applying ELECTRE III, in particular, concerning preference information: if the decision maker's cannot provide precise and complete weight information, or if there are multiple decision maker's with conflicting preferences, ELECTRE methods cannot be used for decision process. Wang and Triantaphyllou (2004, 2008) and Wang, (2007) tested ELECTRE II and ELECTRE III methods, and withdrew the same conclusions as Kangas *et al.* (2001) and Tervonen *et al.* (2005).

According to the above, it is important not to focus only on one method, but to analyze the decision problem considering other methods, taking into account the specific characteristics of each one. So, in the future, portfolio management decision problem can also be solved using, for instance, PROMETHEE family or AHP method, presenting themselves as successful alternatives rather referred in literature.

BIBLIOGRAPHIC REFERENCES

Afshari, R., Mojahed, M., Yusuff, M., Hong, S. and Ismail, M.Y. (2010). "Personnel selection using ELECTRE". *Journal Applied Science*, 10: 3068-3075.

Belton V. and Stewart TJ. (2001). "*Chapter 8: outranking methods. In: Multiple criteria decision analysis: an integrated approach*". Boston, MA, USA: Kluwer Academic Publishers.

Brans, J. P. and Vincke (1985). "A preference ranking organization method: The PROMETHEE method". *Management Science* 31,647–656.

Bregar, A., Györkös, J. and Juric, M. (2009), "Robustness and Visualization of Decision Models", *Informatica* 33 (2009) 385–395.

Buchanan, Sheppard and Vanderpooten (1999). "Project ranking using electre III". Presented at *MCDM 2000*, Ankara, Turkey.

Coelho, D., Antunes, H., and Martins, G. (2009). "Using SSM for Structuring an MCDA Model for Sustainable Urban Energy Planning". 5th International Vilnius Conference, EURO Mini Conference, "Knowledge-Based Technologies and OR Methodologies for Strategic Decisions of Sustainable Development" (KORSD-2009), September 30–October 3, 2009, Vilnius, LITHUANIA, ISBN 978-9955-28-482-6.

Cohen, K. and Jerry P. (1967). "An Empirical Evaluation of Alternative Portfolio Selection Models". *Journal of Business* 40(2), pp.166-193.

Damart, S., Dias, L., and Mousseau, V. (2007). "Supporting groups in sorting decisions: Methodology and use of a multi-criteria aggregation/disaggregation DSS". *Decision Support Systems* 43:4 (2007) 1464-1475.

Dias, L., Mousseau, V., Figueira, J., and Clímaco, J. (2002). "An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI". *European Journal of Operational Research*. 138:2 (2002) 332-348, http://hdl.handle.net/10316/5503.

Elton, J., Gruber, J., and Padberg, W. (1976). "Simple Criteria for Optimal Portfolio Selection". *Journal of Finance*, 31(5), pp.1341-1357.

Escobar-Toledo and López-García (2005). "The use of multicriteria decision aid system in the information technology (It) allocation problem". *Operational Research* 5(2): 223-240.

Fama, E. (1965). "The Behavior of Stock Market Prices". *Journal of Business.*

Figueira, J. and Roy, B. (2002). "Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure". *European Journal of Operational Research*, 139:317–326, 2002.

Figueira, J., Mousseau, V., and Roy, B. (2005). "*Chapter 1: Electre Methods*". Multiple Criteria Decision Analysis: State of the Art Surveys, Springer's International Series in Operations Research & Management Science, 2005, Volume 78, III, 133-153.

Georgopoulou, E., Lalas, D. and Papagiannakis, L. (1997). "A multicriteria decision aid approach for energy planning problems: The case of renewable energy option". *European Journal of Operational Research* 103, 38–54.

Hanandeh, A. and El-Zein, A. (2006). "A new stochastic multi-criteria decision analysis tool base on ELECTRE III method". School of Civil Engineering, The University of Sifney.

Hokkanen, J. and Salminen, P. (1997). "Electre III and IV decision aids in an environmental problem". *Journal of Multi-criteria Decision Analysis* 6: 215–26.

Huang, W. and Chen, C. (2005). "Using the Electre II method to apply and analyze the differentiation theory ". proceedings of the Eastern Asia Society for Transportation Studies, Vol. 5, pp. 2237 - 2249.

Hwang, C-L and Yoon, K. (1981). "*Multiple Attribute Decision Making: Methods and Applications*". Berlin, Springer-Verlag.

Kangas, A., Kangas, J. and Pykäläinen, J. (2001). "Outranking methods as tools in strategic natural resources planning". *Silva Fennica* 35(2): 215–227.

Karagiannidis, A. and Moussiopoulos, N. (1997). "Application of ELECTRE III for the integrated management of municipal solid wastes in the greater Athens area". *European Journal of Operational Research* 97 (3), 439–449.

Kogan, N. and Wallach, M. (1967). "*Risk Taking as a Function of The Situation, The Person and the Group*". In New Directions in Psychology, III (G. Mandler, Ed.). Holt, Rinehart & Winston, New York.

Lahdelma, R. and Salminen, P. (2004). "Stochastic multicriteria acceptability analysis using the data envelopment model". *European Journal of Operational Research*.

Lee, J., Kim, H. and Chu, S. (1989). "Intelligent Stock Portfolio Management System". *Expert Systems* 6(2), pp.74-87.

Lintner, J. (1965). "The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets". *Review of Economics and Statistics* 47, 13-37.

Markowitz, H. (1952). "Portfolio selection". *Journal of Finance*, 7(1):77-91.

Markowitz, H. (1959). "*Portfolio Selection: Efficient Diversification of Investments*". New York: John Wiley & Sons. http://cowles.econ.yale.edu/P/cm/m16/index.htm. (reprinted by Yale University Press, 1970, ISBN 978-0300013726; 2nd ed. Basil Blackwell, 1991, ISBN 978-1557861085).

Maystre, L., Pictet, J., and Simos, J. (1994). "Les Methodes Multicrit`eres ELECTRE". *Presses Polytechniques et Universitaires Romandes, Lausanne.*

Mousseau, V. (1995). "*Eliciting information concerning the relative importance of criteria*". In: Pardalos, Y., Siskos, C., Zopounidis, C. (Eds), Advances in multicriteria analysis. Kluwer academic publishers, Dordrecht, pp. 17-43.

Miller, G. (1978)."Properties of Certain Symmetric Stable Distributions". *Journal of Multivariate Analysis*, 8, pp. 346-360.

Mossin (1966)."Equilibrium in a Capital Asset Market". *Econometrica*, 34(4), pp.768-783.

Rogers, M. and Bruen, M. (1996). "Using ELECTRE to rank options within a highway environmental appraisal two case studies". *Civil Engineering Systems* 13:203–21.

Rogers, M. and Bruen, M. (1998a). "Choosing realistic values of indifference, preference and veto theresholds for use with environmental crireia within ELECTRE". *European Journal of Operacional Research*, 107(3): 542-551.

Rogers, M. and Bruen, M. (1998b). "New system for weighting environmental criteria for use within ELECTRE III". *European Journal of Operacional Research*, 107(3): 552-563.

Rogers, M., Bruen, M. and Maystre L. (1999). "*Chapter 3: the electre methodology. Electre and decision support*". Boston, MA, USA: Kluwer Academic Publishers.

Rogers, M., Bruen, M. and Maystre L. (1999). "*Chapter 5: case study 1: finding the best location for the galway wastewater treatment plant. Electre and decision support*". Boston, MA, USA: Kluwer Academic Publishers.

Rogers, M., Bruen, M. and Maystre L. (1999). "*Chapter 6: case study 2: choosing the best waste incineration strategy for the eastern Switzerland region. Electre and decision support*". Boston, MA, USA: Kluwer Academic Publishers.

Roy (1991). "The outranking approach and thinks of ELECTRE methods". *Theory and Decision*, 31:49-73.

Roy B. and Bouyssou D. (1993), Aide Multicritère à la Décision: Méthodes et Cas, *Édition Économica*, Paris.

Schärlig, A. (1985). "*Décider sur plusieurs critères - Panorama de l'aide à la décision multicritère*", Presses Polytechniques Romandes, Lausanne.

Saaty, T. (1980). "*The analytical hierarchy process*". McGraw-Hill: New York.

Salminen, P., Hokkanen, J. and Lahdelma, R. (1998). "Comparing multicriteria methods in the context of environmental problems". *European Journal of Operational Research* 104: 485–496.

Serdar, U. **a**nd Aynur, K. (2009). "A multiple criteria decision-making approach to the selection of concrete pumps". *Journal of Civil Engineering and Management*, Volume 15, Issue 4, pp. 369-376.

Shane, B., Fry, M. and Toro, R. (1987). "The Design of an Investment Portfolio Selection Decision Support System Using Two Expert Systems and a Consulting System". *Journal of Management Information Systems* 3(4), pp.79-92.

Slovic, P. (1964). "Information Processing, Situation Specify and the Generality of Risk-Taking Behavior", *J. Pers. Soc. Psychol.* 22,pp.128-134 .

Sharpe, W. (1964). "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk". *Journal of Finance*, 19(3), pp.425-442.

Sobral M., Costa A. P., and Filho A. (2010), "Multi-criteria Model for Classifying Clients of Companies Distributing Liquefied Petroleum Gas", Brazilian Journal of Operations & Production Management Volume 7, Number 1, 2010, pp. 53-75.

Steuer, Ralph E., and Na, P. (2003), "Multiple criteria decision making combined with finance: A categorized bibliographic study", European Journal of Operational Research, 2003, vol. 150, issue 3, pages 496-515.

Tervonen, T., Figueira, J., Lahdelma, R., and Salminen, P., (2005), "An Inverse Approach for ELECTRE III", in proceedings of the 61th meeting of the EURO EWG, Luxembourg, March, 2005.

Tobin, J. (1958). "Liquidity preference as a behaviour toward risk". *Review of Economic Studies* 25: 65-86.

Triantaphyllou, E. (2000). "Multi criteria decision making methods: A comparative study". *Kluwer Academic Publishers*, Boston, MA, USA.

Wang and Triantaphyllou, E., (2004), "*Some Ranking Irregularities When the ELECTRE Method is Used for Decision-Making*," Proceedings of the 2004 IIE Annual Conference, May 2004, Houston, TX.
Wang and Triantaphyllou, E., (2008), "*Ranking Irregularities When Evaluating Alternatives by Using Some ELECTRE Methods*", *Omega*, Vol. 36, pp. 45-63.

Wang, (2007), "*Study of ranking irregularities when evaluating alternatives by using some ELECTRE methods and a proposed new MCDM method based on regret and rejoicing*", thesis submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the Master of Science in Industrial Engineering.

Xidonas, P., Askounis, D., and Psarras, J. (2009), "Common stock portfolio selection: a multiple criteria decision making methodology and an application to the Athens Stock Exchange", Oper Res Int J (2009) 9:55–79, DOI 10.1007/s12351-008-0027-1.

ZAK, J. (2005), "The Comparison of multiobjective ranking methods applied to solve the mass transit systems decision problems", e-proceedings of the 16th Mini - EURO Conference and 10th Meeting of EWGT.

Zbigniew, P. and Watróbski, J. (2008). "Environmental factors as determinants of multi criteria methods suitability for a decision situation". Szczecin University of Technology, Faculty of Computer Science and Information Technology, Metody Informatyki Stosowanej, nr 2/2008.

Zopounidis, C. (1992), "On The Use of The Minora Multi criteria Decision Aiding System To Portfolio Selection and Management", Working paper 92-07 (June 1992), Department of Production Engineering and Management, Technical University of Crete.

Zopounidis, C., D.K. Despotis and I. Kamaratoy (1993), "Portfolio Selection Using The Adelais Multiobjective Linear Programming System", Working paper 93-10 (October 1993), Department of Production Engineering and Management, Technical University of Crete.

CIGE – Centro de Investigação em Gestão e Economia
Universidade Portucalense – Infante D. Henrique
Rua Dr. António Bernardino de Almeida, 541/619
4200-072 PORTO
PORTUGAL

http://www.upt.pt
cige@uportu.pt

ISSN 1646-8953